P1 Chapter 7 :: Algebraic Methods

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Last modified: 16th September 2018

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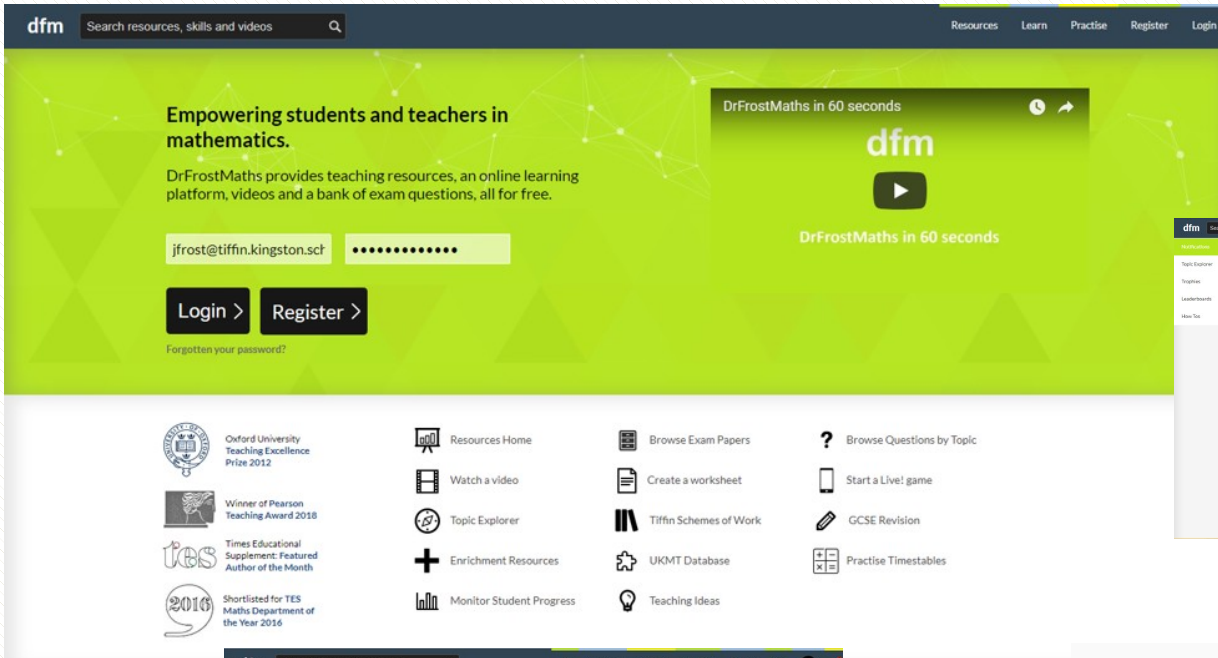

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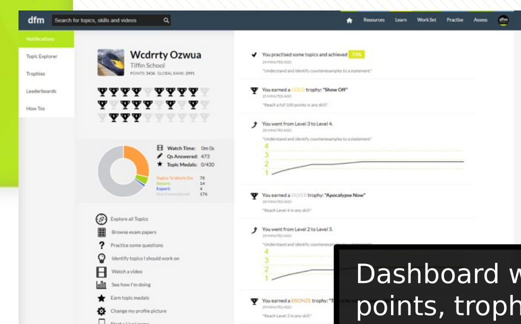

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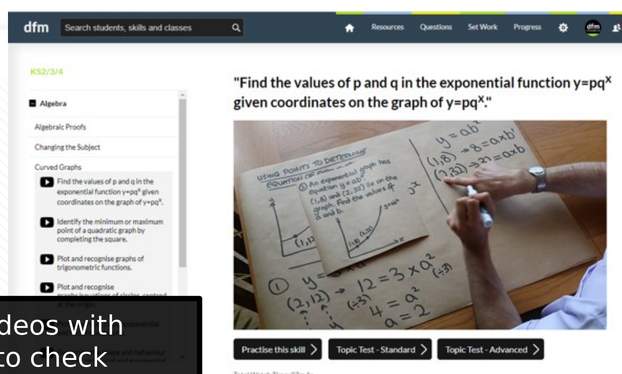
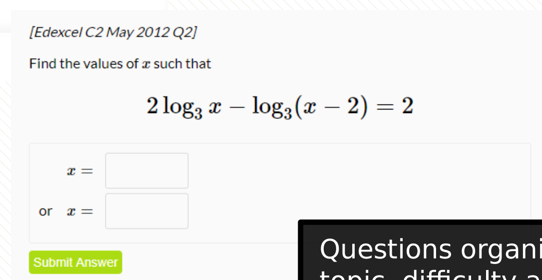


Teaching videos with topic tests to check understanding.



Questions organised by topic, difficulty and past paper.

Chapter Overview

This chapter covers a number of algebraic techniques, but also algebraic ‘proofs’.

1:: Algebraic Fractions

Simplify

2:: Dividing Polynomials

Divide

3:: The Factor Theorem

Given that is a factor of , find the value of .

4:: Proof

Prove that all square numbers are either a multiple of 4 or 1 more than a multiple of 4.

A Level 2017 specification note: The ‘Remainder Theorem’ has been removed (it was included in C2). Sad times…

1 :: Simplifying Algebraic Fractions

Recall that you can simplify fractions by **dividing** the numerator and denominator by a **commo**

Fro Hint: To identify common factors we need to factorise first.

$$\frac{x^2-1}{x^2+x}=$$

?

Fro Note: Do not leave 1 in the denominator!

$$\frac{x^2+3x+2}{x+1}=$$

?

Fro Tip: Factorise the easier one first because it provides clues to the ther.

$$\frac{2x^2+11x+12}{x^2+9x+20}=$$

?

$$\frac{4-x^2}{x^2+2x-8}=$$

?

Fro Tip:

Exercise 7A

Pearson Pure Mathematics Year 1/AS
Pages 138-139

Terminology

$$11 \div 4 = 2\, rem\, 3$$

?

(the thing we're dividing)

?

?

t

?

er

Normal Long Division

```
      38.
11 | 423.00
     33
     0 0
      9
     38
     8  5  0
```

1. We found how many whole number of times (i.e. the quotient) the divisor went into the dividend.

2. We multiplied the quotient by the dividend.

3. …in order to find the remainder.

4. Find we 'brought down' the next number.

How many times does go into ?

Now repeat! How many times does go into ?

$$6^2 \;-\; 2 \;+\; 3$$

$$+\;5 \;\big)\; 6^3 + 28^2 - 7 + 15$$

Multiply by . The first term should match with above.

$$6^3 + 30^2$$

Subtract and carry down next term.

$$-\;2^2 - 7$$

$$-\;2^2 - 10$$

$$3 \;+$$

$$35+$$

$$15 \qquad 0$$

This is the remainde

Fro Tip:
You can check your solution by expanding:

But if you know you should get no remainder, ending with 0 at the bottom is a good

Fro Tip:
Be ***very*** careful subtracting negatives.

Further Example

Find the remainder when is divided by .

$$3^2 \quad + \qquad +1$$

$$-\,1 \;\Big)\; 3^3 \;\; 3 + \;\; 0^2 - 2 \;\; + 4$$

$$3^3 - \quad 3^2$$

$$3x^2 - 3x$$

$$x + 4$$

$$x - 1$$

$$5$$

Fro Note: Ensure that any missing terms in the polynomial are filled in, so that the powers decrease by 1 each time.

The remainder is 5

Test Your Understanding

Find the remainder when is divided by .

?

6

Further Test Your Understanding

Divide by .

?

Exercise 7B

Pearson Pure Mathematics Year 1/AS
Pages 141-142

3 :: The Factor Theorem

$$x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

We can see that is a factor of .
What would happen if is 2?

?

then would be a factor.

! The Factor Theorem states that if is a polynomial then:

- If , then is a factor of .
- Conversely, if is a factor of , then .

Examples (in textbook)

Show that is a factor of .

?

Fully factorise .

?

algebraic division.

Using Factor Theorem to find unknown coefficients

Given that is a factor of , determine the value of .

?

Test Your Understanding

Edexcel C2 May 2016 Q2

) $= 6x^3 + 13x^2 - 4$

No long in spec.

~~(a) Use the remainder theorem to find the remainder when f(x) is divided by ($2x + 3$).~~ ~~**(2)**~~

(b) Use the factor theorem to show that ($x + 2$) is a factor of f(x). **(2)**

(c) Factorise f(x) completely. **(4)**

?

Given that is a factor of , determine the value of .

?

Exercise 7C

Pearson Pure Mathematics Year 1/AS
Pages 145-146

Extension

1 *[MAT 2006 1E]* The cubic

Has both and as factors.
Determine the values of and .

?

2 [MAT 2009 1I] The polynomial

Has as a factor
A) for no values of ;
B) for only;
C) for only;
D) for and only.

?

option B.

The **remainder theorem** states that if is divided by , the remainder is .
This similarly works whenever makes the divisor 0.

3 [MAT 2013 1G] Let be an integer and be the polynomial

What is the remainder, in terms of , when is divided by ?

?

4 :: Proof

Terminology

A **conjecture** is a mathematical statement that has yet to be proven.

One famous conjecture is **Goldbach's Conjecture**. It states "*Every even integer greater than 2 can be expressed as the sum of two primes*." It has been verified up to (that's big!); this provides evidence that it is true, but does not prove it is true!

A **theorem** is a mathematical statement that has been proven.

One famous misnomer was **Fermat's Last Theorem**, which states "*If* is an integer where *, then has no non-zero integer solutions for* ." It was 300 years until this was proven in 1995. Only then was the 'Theorem' in the name then correct!

Types of Proof

A proof must show all **assumptions** you are using, have a clear **sequential list of steps** that logically follow, and must cover **all possible cases**.
You should usually make a **concluding statement**, e.g. restating the original conjecture that you have proven.

a. Proof by Deduction

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

"Prove that the product of two odd numbers is odd."

?

An **identity** is an equation that is true for **all values** of the variables. e.g. is true only for , but is true for all .

"Prove that "

?

Be Warned...

Proof by Deduction requires you to **start from known facts** and end up at the conclusion. It is **not** acceptable to start with to the conclusion, and verify it works, **because you are assuming the thing you are trying to prove**.

Example: Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5.

Incorrect Proof:

?

We are assuming the thing we are trying to p

Correct Proof:

?

We are only assuming things in the 'if' bit. This is fine!

The underlying problem is that this technique doesn't prove there can't be **other** consecutive integers that work – we have only verified 3,4,5 is one such solution

Types of Proof

a. Proof by Deduction

Prove that is positive for all values of .

?

Fro Exam Tip: This is quite a common last parter.

Anything squared is at least 0. This is formally known as the '*trivial inequality*'.

Test Your Understanding

Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.

?

which is 2 more than a multiple of 8.

Exercise 7D

Pearson Pure Mathematics Year 1/AS
Pages 149-150

Extension

[STEP I 2005 Q1] 47231 is a five-digit number whose digits sum to
.

(i) Prove that there are 15 five-digit numbers whose digits sum to 43. You should explain your reasoning clearly.

(ii) How many five-digit numbers are there whose digits sum to 39?

? i

possibilities.

? ii

Other Types of Proof

b. Proof by Exhaustion

This means breaking down the statement into **all possible smaller cases**, where we prove each individual case.

(This technique is sometimes known as 'case analysis')

Prove that is even for all integers .

?

c. Disproof by Counter-Example

While to prove a statement is true, we need to prove every possible case (potentially infinitely many!), **we only need one example to disprove** a statement.

This is known as a **counterexample**.

Disprove the statement:
" is prime for all integers ."

?

Exercise 7E

Pearson Pure Mathematics Year 1/AS
Pages 152-154